Laplacian-Gaussian Sub-Correlation Analysis for the Extraction of Maneuvering Affordance in Random Image Fields

Kohji Kamejima\textsuperscript{2}

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Abstract: Scale shift of random texture due to perspective projection is available as the cue to estimate the expansion of almost horizontal space. To detect the image of possible routes as a version of maneuvering affordance, in this paper, two universal imaging rules, perspective projection and self-similarity, are combined. By identifying scale feature to invariant measure associated with a fractal attractor, the expansion of maneuvering area is estimated. The detection scheme was verified through experimental studies.

Keywords: Maneuvering Affordance; Perspective Projection; Self-Similarity; Scale Feature

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\textsuperscript{2}Professor emeritus, Osaka Institute of Technology, kamejima@k.zaq.jp
1 Introductory Remarks

Visual perception organizes randomly distributed image features into cues to subsequent decisions. As an as-is part of real world, natural environment itself maintains consistency independent of observers. One of the essential capabilities of ‘real world intelligence’, hence, is to incubate propositional goal within the image of the scene as the basis of the belief on real world. Such intentional image analysis takes a crucial role in various implementation of end user programming [4], knowledge reuse [11], the open logic machine [10], driver’s associate [2] and explorative education [1], e.g. Following ecological optics [3], in fact, perception process is supported by the optical array filling natural scene. The information conveyed by such ecological optics transfers the implications of environment as the affordance: the message to be captured for inducing plausible decisions. In this paper, a computational scheme is presented for extracting maneuvering affordance [8] based on two types of universal rules: perspective projection and self-similarity.

2 Generic Roadway Model

Consider the image of a scene observed by a pedestrian as shown in Fig. 1 where the distribution of objects is modulated on the brightness distribution. Through the image, the pedestrian is required to estimate the expansion of nearly horizontal plane from current position as the support of possible maneuvering.

On the earth, generally, the direction of the gravity force induces a natural 3D interpretation rule in image plane. Let $\Omega \subset \mathbb{R}^2$ be a fixed image plane with pixel $\omega = (x, y) \in \Omega$, $x \in [-N_X, N_X]$, $y \in [0, N_Y]$ and consider 3D coordinate system in which the image plane $\Omega$ is associated with the intention of motion as illustrated in Fig. 2. Through the intentional coordinate system, Depth information is coordinated along y-axis of image plane $\Omega$. This implies that perspective projection, a universal rule, induces generic
image of the intention for maneuvering on the image plane \( \Omega \). For instance, random texture at the foot with scale components as indicated in Fig. 3 yields a directional Fourier image \([9]\) as illustrated in Fig. 4. As shown in figure 4, the directional Fourier image indicates the maneuvering affordance without explicit geometric information.

In other words, however, the information conveyed by the directional Fourier image cannot be matched with the brightness distribution in the image plane \( \Omega \). To match the existence information with scene geometry, in what follows, the support of the maneuvering is assumed to be generated computationally through the following self-similarity processes:

\[
\square \implies \square \implies \square.
\]

The dynamics (1) is known to generate a fractal attractor \( \Xi \) with sufficient randomness as illustrated in Fig 5. Thus, we can adopt the self-similarity process (1) as the second universal rule to generate a generic model for the top-view of a street. Let \( \nu = \{ \mu_i, i = 1, 2, \ldots, m \} \) be the set of contraction mappings governing the process (1). Giving the mapping set \( \nu \), the probability for capturing the attractor \( \Xi \) in image plane \( \Omega \) is represented by the solution \( \varphi(\omega|\nu) \) to the following equation \([6]\):

\[
\frac{1}{2} \Delta \varphi(\omega|\nu) + \rho [\chi_\Xi(\omega) - \varphi(\omega|\nu)] = 0,
\]

where \( \chi_\Xi \) denotes the invariant measure associated with the attractor \( \Xi \) and \( \rho = \log m \) is the complexity parameter. Noticing the following association

\[
\frac{1}{\rho} \frac{d\varphi(\omega|\nu)}{d\tau} \sim \frac{\varphi(\omega|\nu) - \chi_\Xi(\omega)}{1/\rho} = \frac{1}{2} \Delta \varphi(\omega|\nu),
\]
for $\tau \sim 1/\rho$, the distribution $\varphi(\omega|\nu)$ is visualized as the image consisting of the components with scale $\sigma$, $0 < \sigma \leq \tau$. In many practical situations, the invariant measure $\chi_\Xi^P$ is obscured by background noise as illustrated in Fig. 6. Even in such noisy imagery, the equation (2) yields a sufficiently smooth field as shown in Fig. 7. By definition of $\varphi(\omega|\nu)$, we can specify the most probable points in the attractor $\Xi$ as the following local maxima:

$$\tilde{\Theta} = \{ \tilde{\theta} \in \Lambda \mid \nabla \varphi = 0, \det [\nabla \nabla^T \varphi] > 0, \Delta \varphi < 0 \}.$$  

Noticing the stochastic-computational detectability of the self-similarity [7], we can verify the existence of the attractor $\Xi$ on discrete set $\tilde{\Theta}$ and discriminate generic roadway model from noisy background as shown in Fig. 8. Hence, the application of the second universal rule results in the transformation of the brightness distribution $f(\omega)$ into invariant measure $\chi_\Xi^P$.

### 3 2D Laplacian-Gaussian Analysis

The top view of the generic roadway model generated via the second universal rule (1) is projected on the image plane $\Omega$. The image of the generic model is skewed by the perspective projection. Following the first universal rule as illustrated in Fig. 4, the image of the generic model can be featurized in terms of scale shift.
Consider an image of exact point located at $\xi \in \Omega$. In many imaging devices including human eye, the point image $\delta_\xi$ is transformed to smooth gray level distribution through the following Gaussian point spread function

$$g_\sigma(\omega|\xi) = \frac{1}{2\pi\sigma^2} \exp \left[ -\frac{|\omega - \xi|^2}{2\sigma^2} \right],$$

where $\sigma$ denotes the scale parameter. Noticing

$$\sigma^2 \left[ \frac{1}{2} \Delta g_\sigma(\tilde{\theta}|\xi) \right] = g_\sigma(\tilde{\theta}|\xi),$$

(3)

at local maximum point $\tilde{\theta}$ of $g_\sigma$, we can estimate the scale parameter $\sigma$ based on observables $\Delta g_\sigma(\tilde{\theta}|\xi)$ and $g_\sigma(\tilde{\theta}|\xi)$ without explicit information $\xi$. Generally, consider the following decomposition

$$f_\sigma(\omega) \sim g_\sigma * \chi^P_\Omega(\omega) + b(\omega),$$

(4a)

where $b(\omega)$ denotes nonnegative bias satisfying

$$|\Delta b| \ll |\Delta g_\sigma|.$$  

(4b)

Noting biased correlation $\sigma^2 \left[ \frac{1}{2} \Delta f(\tilde{\theta}) \right] \sim f(\tilde{\theta}) - b(\tilde{\theta})$, we have

**Proposition 1** (Sub-Correlation) The upper bound for scale parameter $\sigma$ at $\tilde{\theta}$ can be estimated by

$$\sigma^2(\tilde{\theta}) \leq \frac{f(\tilde{\theta})}{\frac{1}{2} \Delta f(\tilde{\theta})},$$

(5)

at each local maximum point $\tilde{\theta}$ of the imagery $f(\omega)$, $\omega \in \Omega$.

The weak constraint (5) can be extracted via local analysis based on $f$ and $\Delta f$.

Consider the following representation of scale component

$$f_\sigma(\omega) = |f_\sigma| \exp \left[ 2\pi j \frac{x + y}{\sigma} \right],$$

(6)

for $\omega = (x, y)$. For small scale deviation $\varepsilon$, it follows that

$$\frac{f_{\sigma+\varepsilon}(\omega)}{f_\sigma(\omega)} = \Re \left[ \exp \left[ 2\pi j \frac{\varepsilon(x + y)}{\sigma(\sigma + \varepsilon)} \right] \right],$$

where $\Re[\cdot]$ denotes the real part of complex number $(\cdot)$. Noting that

$$\Re \left[ \exp \left[ 2\pi j \frac{\varepsilon(x + y)}{\sigma(\sigma + \varepsilon)} \right] \right] d\omega \sim \frac{1}{2\pi} \exp \left[ -\frac{\varepsilon^2}{2} \right] d\omega,$$

for $\omega = (x, y)$. For small scale deviation $\varepsilon$, it follows that

$$\frac{f_{\sigma+\varepsilon}(\omega)}{f_\sigma(\omega)} = \Re \left[ \exp \left[ 2\pi j \frac{\varepsilon(x + y)}{\sigma(\sigma + \varepsilon)} \right] \right],$$

where $\Re[\cdot]$ denotes the real part of complex number $(\cdot)$. Noting that
with scale deviation $\varepsilon$ can be indexed as a measure in $\Omega$ in terms of the following normal distribution

$$n_\sigma(\varepsilon) = \frac{1}{\sqrt{2\pi}} \exp\left[\frac{-\varepsilon^2}{2}\right].$$

(7)

Hence we have the measure indicating the probability for detecting $\sigma$-scale component. The scale space feature $n_\sigma(\varepsilon)$ generated by the scene image is shown in Fig. 9 where the scale parameter is shifted in accordance with perspective projection

$$\sigma(y) = \frac{\sigma_0}{y_\infty - y_0} (y_\infty - y).$$

(8)

with $y_\infty = N_y$. In eq. (8), $\sigma_0$ is adjusted to the dominant scale of $\Delta f(x, y_0)$. By identifying

$$\hat{\chi}(\omega) = n_\sigma(y)f,$$

(9)

we have the capturing probability for underlying fractal attractor as the solution to the following diffusion system:

$$\frac{1}{2} \Delta \varphi_y(\omega|\nu) + \rho[\hat{\chi}(\omega) - \varphi_y(\omega|\nu)] = 0.$$  

(10)

Thus, the most probable points of not-yet-identified attractor $\Xi$ can be detected by the following local maxima:

$$\tilde{\Theta} = \left\{ \tilde{\theta} \in \Lambda \mid \nabla \varphi_y = 0, \det [\nabla \nabla^T \varphi_y] > 0, \Delta \varphi_y < 0 \right\}.$$  

4 Fractal Feature Estimation

Perspective projection induces the proportionality (8). This implies that almost horizontal plane is confined in a subset of image plane $\Omega$, called the support. The support of the proportionality is visualized as a fractal attractor with fixed points

$$\Omega_f^\nu = \{ \omega_\mu^f, \mu_i \in \nu \},$$

as illustrated in Fig. 10 [5]. By combining finite sampling $\tilde{\Theta}$, hence, the detection of maneuvering affordance $\Xi$ results in the estimation of the point set $\Omega_f^\nu$ based on the following one parameter family of linear point distribution

$$\tilde{\Theta}_x = \left\{ \tilde{\Theta}_x(y) \mid y_0 \leq y \leq y_\infty \right\},$$

(11a)

$$\tilde{\Theta}_x(y) = \left\{ \tilde{\theta} \mid (\tilde{\theta}, y) \in \tilde{\Theta} \right\}.$$  

(11b)

Define one parameter family of distributions on $\tilde{\Theta}_x$:...
\[ p_y(\tilde{\theta}_x) = C \cdot \varphi((\tilde{\theta}_x, y)|\tilde{\nu}) f(\tilde{\theta}_x, y), \quad (12a) \]

where \( C \) is the constant such that
\[ \sum_{\tilde{\theta}_x \in \tilde{\Theta}_x} p_y(\tilde{\theta}_x) = 1. \quad (12b) \]

By identifying the data \((\tilde{\Theta}_x, p_y)\) with the observation of Gaussian random variables, we have the following statistical moment for estimating the expansion of the maneuvering affordance \( \Xi \)
\[
\begin{align*}
\hat{m}_x(y) & = \sum_{\tilde{\theta}_x \in \tilde{\Theta}_x(y)} \tilde{\theta}_x p_y(\tilde{\theta}_x), \quad (13a) \\
\hat{\sigma}_x^2(y) & = \sum_{\tilde{\theta}_x \in \tilde{\Theta}_x(y)} (\tilde{\theta}_x - \hat{m}_x(y))^2 p_y(\tilde{\theta}_x). \quad (13b)
\end{align*}
\]

By using the one parameter family of the line moments \((m_x^\hat{y}(y), 2\sigma_x^\hat{y}(y))\), the fractal feature is specified as follows:
\[
\begin{align*}
\omega_{\mu_0}^f & = (x_\infty, y_\infty), \quad (14a) \\
\omega_{\mu_1}^f & = (x_0 - 2\sigma_x^\hat{y}(y_0), y_0), \quad (14b) \\
\omega_{\mu_2}^f & = (x_0 + 2\sigma_x^\hat{y}(y_0), y_0), \quad (14c) \\
\omega_{\mu_3}^f & = (x_\infty, -y_\infty). \quad (14d)
\end{align*}
\]

Hence, we have

**Proposition 2 (Statistical Estimate)** The fractal features are specified by solving the following optimization problems:
\[
\begin{align*}
\sum_{\tilde{\theta}_x \in \tilde{\Theta}_x(y)} \left| m_x^\hat{y}(y) - \left( \frac{x_\infty - x_0}{y_\infty - y_0}, y \right) \right|^2 & \rightarrow \min, \quad (15a) \\
\sum_{\tilde{\theta}_x \in \tilde{\Theta}_x(y)} \left| \sigma_x^\hat{y}(y) - \left( \frac{\sigma_0^\hat{y}}{y_\infty - y_0}, y \right) \right|^2 & \rightarrow \min, \quad (15b)
\end{align*}
\]
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Figure 11: Visualized Maneuvering Affordance

\[
\sum_{\tilde{\theta}_x \in \Omega_x(y) \atop y_0 \leq y \leq y_\infty} p_y(\tilde{\theta}_x) \rightarrow \text{max},
\]

where \(\Omega_x(y) = \{\tilde{\theta}_x\}\) is the subset of \(\tilde{\Theta}_x(y)\) satisfying

\[
m_\tilde{x}(y) - 2\sigma_\tilde{x}(y) \leq \tilde{\theta}_x \leq m_\tilde{x}(y) + 2\sigma_\tilde{x}(y).
\]

5 Computational Invariance Test

The optimization problem (15) can be approximately solved by the following monotone procedure:

**Step-1:** Using the initial guess \(y_0 = 0\) and \(y_\infty = N_y\), solve the minimization problem (15a) to obtain the estimate of the center of maneuvering area \((x_0, x_\infty)\).

**Step-2:** Solve the minimization problem (15b) to obtain the estimate of the width of maneuvering area \(\sigma_\tilde{x}\).

**Step-3:** Evaluate (15c) to specify the vanishing point \(y_\infty: y_0 \rightarrow N_y\). Based on the estimates \((x_0, y_0)\) and \((x_\infty, y_\infty)\), we can specify the mapping set as follows:

\[
\hat{\nu} = \{\hat{\mu}_i\}, \quad \hat{\mu}_i(\omega) = \frac{1}{2} [\omega + \omega_{i \mu_i}].
\]

By applying the mapping set to the stochastic features \(\tilde{\Theta}\), we can test the existence of the invariant feature \(\Theta\) through the following finite computation process:

\[
\Theta_{t+1} = \{\theta \in \Theta_t \mid \exists \mu_i \in \nu : \mu_i^{-1}(\theta) \in \Theta_t\},
\]

with initial data \(\Theta_0 = \tilde{\Theta}\).

6 Experiments

The representation scheme was tested via experimental studies. An example of experimental results is shown in Fig. 1, where the reduction scheme (17) was applied to detect invariant subset \(\Theta\) in the stochastic features \(\Theta\). As the result, it was verified that the scene image supports a maneuvering affordance as indicated in Fig. 11. Another example of experimental results is shown in Fig. 12 where a support for possible maneuvering is detected successfully.
7 Concluding Remarks

Perspective projection and self-similarity were combined to detect the maneuvering affordance in complex scene. By evaluating the scale shift due to perspective through Laplacian-Gaussian analysis, the invariant measure associated with unknown fractal attractor is estimated in noisy scene image. A generic roadway model is matched with estimated measure to extract and visualize maneuvering affordance. The representation scheme was verified through experimental studies.

References